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MPAMOD:
AN OPTIMIZATION MODEL
FOR MARITIME PATROL AVIATION
MODERNIZATION PLANNING

by

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March 1993

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This thesis describes an integer programming model to aid in the development of long-range modernization plans for the Navy's Maritime Patrol Aviation fleet. The model is a production/inventory model implemented in the General Algebraic Modeling System (GAMS) and solved with the X-System. Over a user-defined planning horizon, at a yearly level of detail, the model determines an optimal schedule for procuring new aircraft, refurbishing the airframes of existing aircraft through the Sustained Readiness Program and the Service Life Extension Program, upgrading avionics, and retiring old aircraft. Constraints enforce minimum inventory levels, mission effectiveness goals by mission area, goals for minimum average life remaining of the force, budget limitations and annual line capacities for producing, refurbishing and upgrading aircraft. A typical model involves 4,800 constraints and 13,000 general integer variables and is solved to within 4% of optimality in 7.5 minutes on an Amdahl 5990-500.

THESIS DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

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I. INTRODUCTION

The U.S. Navy's Maritime Patrol Aviation (MPA) force is made up entirely of Lockheed P-3 Orion aircraft. The current fleet of P-3s, procured largely in the late 1960s and early 1970s, is well along in its operational service life. Concern over the advancing age of the P-3 inventory has focused the MPA community's attention on modernizing the fleet. Without a modernization program, which may include some mix of refurbishing existing airframes, procuring new aircraft and upgrading the avionics in existing aircraft, there will not be enough aircraft to support the desired number of squadrons past 1997 and the technological advantage MPA has historically enjoyed over its adversaries could be lost. The purpose of this thesis is to develop an optimization model for determining the most cost-effective plan for meeting the future inventory requirements while ensuring that the technological effectiveness of the fleet remains high.

The Navy's MPA force numbers 27 operational VP (fixed-wing patrol) squadrons, 18 active duty and 9 reserve. In addition, there is one training squadron. An inventory of 293 aircraft is needed to support the operation and training commitments while allowing for attrition and aircraft undergoing long-term maintenance [Ref. 1]. The current inventory of 341 aircraft is composed of 94 P-3Bs and 247 P-3Cs. There is currently an excess inventory; however, this is deceiving due to the age of the fleet. The number of aircraft delivered by year is depicted in Figure 1. The large blocks of aircraft procured in the late sixties and early seventies translate into large numbers of aircraft due

for retirement in the next few years. Efforts have already begun to modernize the fleet, but a comprehensive plan which addresses both inventory and high-technology requirements is needed.

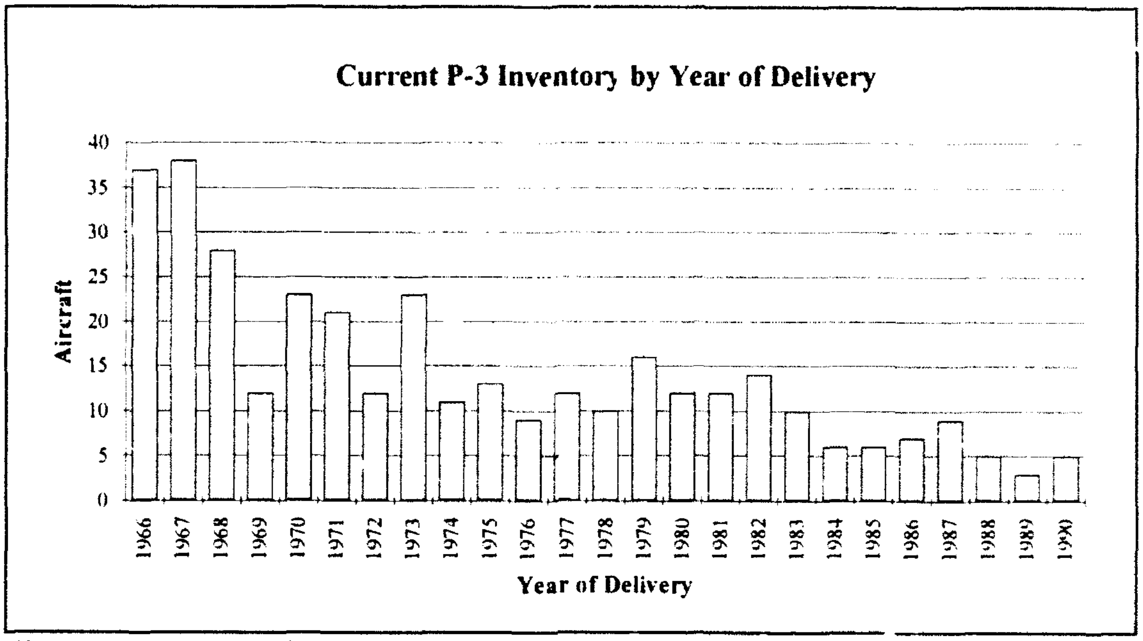

Figure 1 Current P-3 Inventory by Year of Delivery

The MPA modernization problem, which involves not only having sufficient numbers of aircraft but also technologically effective aircraft, is complicated because of the age profile of the fleet and budget limitations. Improving the technological effectiveness of the fleet can be accomplished by updating the avionics systems in existing aircraft with new, high-technology systems; however, attention must be paid to aircraft age. A point will occur in the near future when updated aircraft must be retired and the high-technology advantage will be lost. In many cases the short-term gain in mission performance may not offset the upgrade cost. On the other hand, replacing each retiring aircraft on a one-for-one basis would allow inventory and high technology requirements

to be met but the large number of aircraft due for retirement in the next 10 years makes this prohibitively expensive. The problem facing the MPA community is how to maintain an adequate inventory of aircraft capable of performing the assigned mission while keeping modernization costs within acceptable budgetary limits.

An aircraft is deemed to have reached the end of its operational service life, and must be retired, when it reaches 30 years of age or uses up 100% of its "fatigue life", whichever comes first. Fatigue life is a measure of aircraft life which accounts for the number of landings per flight hour, aircraft usage rate (flight hours per month), types of missions flown, and the environment in which the aircraft is operated. The Fatigue Life Expenditure rate (FLE rate) for aircraft, measured in percent used per 1000 flight hours, has been lower than expected and therefore most aircraft will reach 30 years of age prior to exceeding their fatigue life. Assuming a 30 year life for all aircraft in inventory, 66 aircraft will reach retirement age over the next 5 years making it impossible to support a force structure of 27 squadrons past the year 1997. After 1997, the inventory level falls rapidly to 178 by 2003. Figure 2 depicts the expected inventory level for the next 20 years assuming no modernization effort.

Maintaining 27 VP squadrons will not be possible based on the current inventory level and the projected retirements. One way to ameliorate the problem is to reduce the force structure requirement to meet the expected inventory level. In fact, this has already started: since 1990 the MPA force structure has been reduced from 24 active duty squadrons and 13 reserve to 18 active duty and 9 reserve, with further reductions possible. However, decommissioning squadrons to keep pace with the shrinking

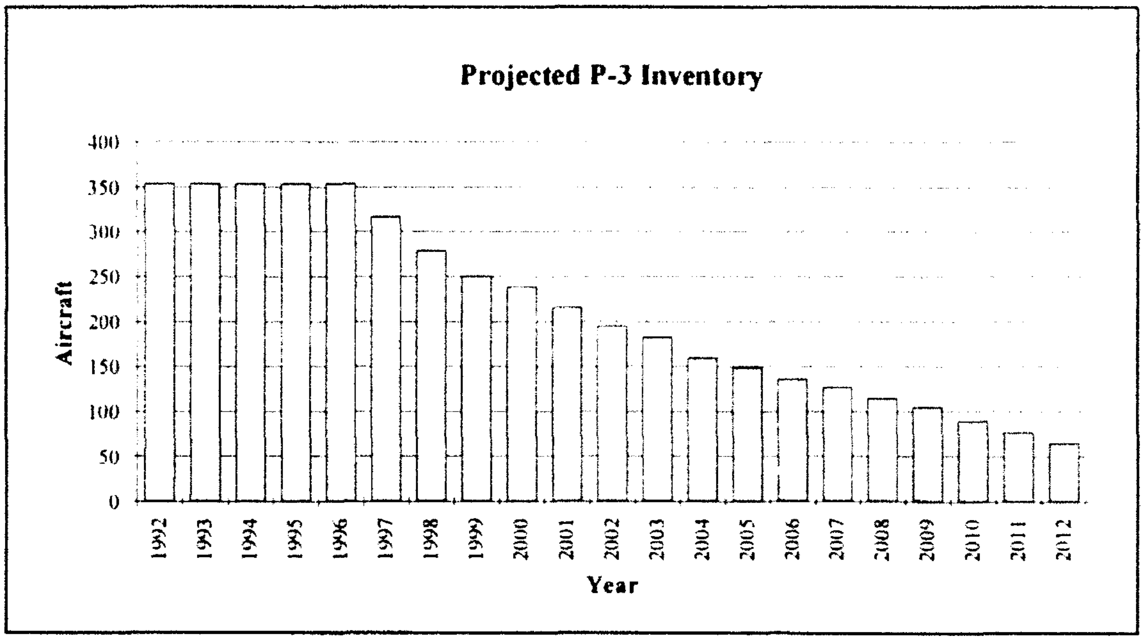

Figure 2 Projected Inventory Level Assuming No Modernization

inventory in this way will only postpone the problem, not solve it. At some point, if a force is to be maintained, action will have to be taken to maintain the inventory level. The inventory level can be maintained partly through the procurement of new aircraft, but currently there is no funded procurement program in place. Given this, the earliest a new aircraft could be seen in inventory is 2001; even then the initial production numbers would be too small to keep pace with the expected P-3C retirements. A more immediate, yet less long-term solution to maintaining the inventory level is to postpone the forthcoming retirements by extending the life of the existing aircraft. There are two maintenance programs designed to extend the airframe life, the Sustained Readiness Program (SRP) and the Service Life Extension Program (SLEP). The SRP, funded to begin in 1996, will allow aircraft to realize 100% of their fatigue life regardless of age. The SLEP, which is still under consideration, is a more extensive program designed to

increase the fatigue life of the aircraft beyond 100%. The final solution to the MPA modernization problem will probably be some combination of reducing the force structure, refurbishing existing aircraft and updating their avionics and procuring new aircraft.

This thesis is concerned with determining the most cost-effective solution to the modernization problem through the use of an optimization model called MPAMOD (MPA MODernization model). The optimization model will determine a schedule for procuring new aircraft, refurbishing the airframes of existing aircraft, modernizing avionics and retiring old aircraft, to meet the Navy's long-range inventory and high-technology goals, while remaining within annual budget limits. The methodology used has precedence in the U.S. Army's helicopter modernization program [Ref. 2], begun in the late 1980s. An optimization model, called "PHOENIX", was developed in support of the helicopter modernization program. The success of the PHOENIX model has led to the extensive use of optimization models by the U.S. Army in support of hardware procurement and upgrades. In addition to the Army's work, a thesis was written in 1990 [Ref. 3] applying the methodology of the PHOENIX model to modernizing the MPA fleet. This model is referred to by its author's name, Drash. The PHOENIX model and the Drash model helped provide a framework for MPAMOD.

A significant number of changes have taken place since the time when the Drash model was completed. An acquisition program, the P-7 program, designed to procure a replacement for the P-3, has been cancelled, and as stated earlier, the required force size has been reduced to 18 active duty and 9 reserve squadrons with further reduction

probable. Furthermore, the Navy's development of the Sustained Readiness Program and Service Life Extension Program adds new alternatives for meeting inventory requirements. These changes in the MPA modernization effort and the fact that the Drash model was never implemented warrant taking a fresh look at modeling the MPA modernization effort.

MPAMOD has many similarities to the PHOENIX and Drash models. All three models establish similar constraints to ensure:

- Required inventory levels are maintained,
- Minimum high-technology requirements are met,
- Maximum average fleet age is not exceeded,
- Expenditures remain within budget limits,
- Minimum and maximum production line limits are not exceeded.

However, there are significant differences between the methodologies used by each model. Both the PHOENIX and Drash models are mixed integer programs (MIPs) with binary and continuous variables. The PHOENIX model uses binary variables to open and close the production facilities while the aircraft updates, SLEPs, procurements and retirements are allowed to take on fractional values. Considering the large number of aircraft being dealt with by the Army, fractional values are acceptable. The Drash model uses similar methods for controlling the opening and closing of a production facility. However, with the small number of aircraft in the MPA fleet, fractional values for updates, retirements and procurement are not acceptable. For aircraft retirements and

updates, the Drash model groups aircraft with similar ages and accumulated flight time into cohorts. Binary variables associated each cohort are used to retire or update the entire cohort at one time. MPAMOD is similar to the Drash model in its use of cohorts, except that general integer variables are used for each cohort rather than binary variables designed to act on the entire cohort at one time. This allows individual aircraft within a cohort to be updated, retired, or sent through SLEP or SRP, independent of one another, while maintaining integrality. The opening and closing year of the production, SRP, SLEP and avionics upgrade facilities are not variable as is the case with PHOENIX and the Drash model, but rather, the opening and closing years are fixed by the user prior to each run.

The drawback to modeling with general integer variables is that solving such a model can be extremely difficult [Ref. 4]. For this reason, two heuristic algorithms are developed for generating near-optimal solutions and to help in obtaining optimal solutions from a general integer solver. The first heuristic solves the model in two phases. The linear program (LP) relaxation is initially solved and then using information from the LP, some of the variables are fixed and certain constraints are removed. The resulting reduced model is then solved as an integer program (IP). The second heuristic employs a rounding technique whereby the integrality requirements are relaxed and the model is successively solved as an LP. The variables within each successive model year are rounded and fixed to integer values prior to the next solve. This rounding of variables, fixing at integer values then resolving continues until the entire planning horizon of the model is covered.

The options available for fleet modernization are described in Chapter II, including descriptions of the avionics upgrade program, SRP, SLEP and the program for procuring a replacement for the P-3. Chapter III describes the mathematical model and its assumptions along with a description of the data needed for the model. The implementation of the model using GAMS [Ref. 5], the X-System solver [Ref. 6] and the heuristics are discussed in Chapter IV along with computational results. Finally, Chapter V contains conclusions and recommendations based on the use of the model.

II. PROBLEM DESCRIPTION

The MPA community is in a serious state of flux. The traditional Anti-Submarine Warfare (ASW) mission, geared mostly toward the former Soviet Union, is changing. Active duty and reserve VP squadrons are being decommissioned, deployment sites are being vacated and stateside bases are being closed. In the midst of this the P-3 is getting old and is in need of structural and avionics modernization. Additionally, the MPA community is facing an inventory shortfall in the late 1990s when the initial block of P-3Cs reaches retirement age. The problem facing the program planners is to reduce the fleet size while economically replacing or refurbishing existing aircraft to develop a modernized fleet capable of performing a more diverse mission. The solution requires replacing the aging aircraft with a mix of new and SRP or SLEP-upgraded aircraft and incorporating avionics upgrades in existing aircraft, while abiding by stringent budget restrictions.

The possible alternatives for modernizing the fleet have been identified by the program planners; what must now be done is to prepare a detailed long-term plan which satisfies possibly conflicting requirements. MPAMOD is formulated as a type of production/inventory model with the modernization requirements implemented as goals rather than hard requirements. MPAMOD expresses these goals in a modest number of constraints with the relative importance of each specified by the program planners. For each year of the planning horizon, a minimum and maximum desired number of aircraft

in inventory is specified, as is a minimum desired average life remaining and minimum desired mission effectiveness by mission area. Financial concerns are expressed through individual budget constraints which limit the annual fixed and unit costs for the production of new aircraft, SRP inductions, SLEP inductions and avionics upgrades. Fixed costs can include Research Development Test & Evaluation (RDT&E) and line opening and closing costs. These costs may begin years before the production starts and end well after the line closes. Unit costs are comprised of the variable costs of the program. Operating and Maintenance (O&M) spending limits are also expressed in budget constraints. In addition to the budget constraints, each program is characterized by minimum and maximum line capacities and cumulative activity limits starting with the line's opening year and continuing through its closing year. This relatively modest number of constraints, addressing mission effectiveness, inventory levels, aircraft age, and facility capacities and budget restrictions captures the essence of an effective modernization program.

A. AIRCRAFT STRUCTURAL UPGRADES

An aircraft's operational service life is affected by two primary factors, corrosion and airframe fatigue. The P-3 was designed in the 1950s to have a 20,000 flight hour fatigue life which, based on the expected annual flight hour usage rate and fatigue life usage rate, equated to about 30 years. What is being found today is that the annual flight hour usage rate is lower than expected and the stress being placed on the aircraft when it is flown is also less than expected. Because of this, aircraft are being stricken from inventory due to structural corrosion problems well before 100% of their fatigue life is

expended. There are two programs in effect targeted at the structural corrosion problems facing the P-3, Standard Depot Level Maintenance (SDLM) and the Sustained Readiness Program (SRP). A third program, the Service Life Extension Program (SLEP) is under consideration.

1. Standard Depot Level Maintenance

An integral part of the effort to control corrosion in the P-3 is the SDLM [Ref. 7]. A majority of the responsibility for corrosion control falls on the individual squadron maintenance departments but depot level maintenance plays a key role in this effort. All P-3 aircraft undergo periodic SDLM which inspects and repairs some corrosion as well as some fatigue problems. The SDLM is not a specialized life-extension program but rather an integral part of the regular P-3 maintenance effort. Aircraft sent to SDLM go through an extensive tear down and inspection, much more detailed than can be done at the squadron level. Visual, x-ray, and ultrasound inspections are performed to identify corrosion and fatigue problems.

SDLM is not unique to the P-3. Virtually every naval aircraft has a SDLM program. The P-3 SDLM program starts in the sixth year of operation with the initial Aircraft Service Period Adjustment inspection (ASPA) [Ref. 8]. An ASPA determines whether there is sufficient corrosion or structural defects to warrant a SDLM. As long as an aircraft passes a yearly ASPA, it may continue to operate. However, once it fails an ASPA, it must undergo a SDLM. After completion of the SDLM, the aircraft is certified safe to fly for a period of 60 months following the first and second SDLM, 50 months for the third and 46 months for subsequent SDLMs [Ref. 3]. After this period,

the ASPA process starts again. Each SDLM varies in cost, depending on the amount of repairs required. However, the costs generally increase for successive SDLMs.

The ASPA inspection adds a stochastic twist to the problem of modeling the SDLM. To get around this, an "average" SDLM schedule is used in MPAMOD to determine when aircraft enter SDLM and at what cost. Historical data is used to determine the average age of aircraft when they undergo each successive SDLM and the average cost of the SDLM. When an aircraft in inventory reaches one of the "average SDLM ages" it incurs the cost for that SDLM.

2. Sustained Readiness Program

The SDLM program cannot guarantee to take an aircraft out to 100% of its fatigue life. There are many critical areas in the P-3 which are inaccessible under the financial and physical constraints of the SDLM program. Corrosion in those areas can force the retirement of an aircraft before its fatigue life is reached. The SRP is designed to correct corrosion and fatigue in those critical areas and allow an aircraft to extend its life beyond the 30 year limit to 100% of fatigue life. The SRP will preemptively repair and replace critical components while it is still economical to repair the aircraft and before corrosion problems begin affecting operational availability. Due to the nature of the program, it does not make sense to induct an aircraft too early in its life thereby preemptively replacing parts which still have a significant amount of life remaining. Conversely, waiting too long to induct an aircraft could result in excessively high repair costs. Therefore, program planners have established an "induction window" of 25 to 29 years of age for identifying candidate SRP aircraft.

Figure 3 depicts expected inventory levels for the next 20 years assuming retirement at 30 years of age with no modernization versus the expected inventory levels if every P-3C achieves 100% of its fatigue life. The significantly increased inventory levels are those that could be achieved by implementing the SRP with an unlimited budget. MPAMOD's estimate of aircraft fatigue life comes from data found in the P-3 Aircraft Structural Appraisal of Fatigue Effects (SAFE) quarterly report [Ref. 9]. Fatigue life remaining, measured as percent life remaining, and fatigue life expenditure rate, measured in percentage of life used per 1000 flight hours, are converted to years of life remaining assuming an nominal P-3 usage of 600 flight hours per year. It is also assumed that the fatigue life expenditure rate for an aircraft does not change over the planning horizon of the model.

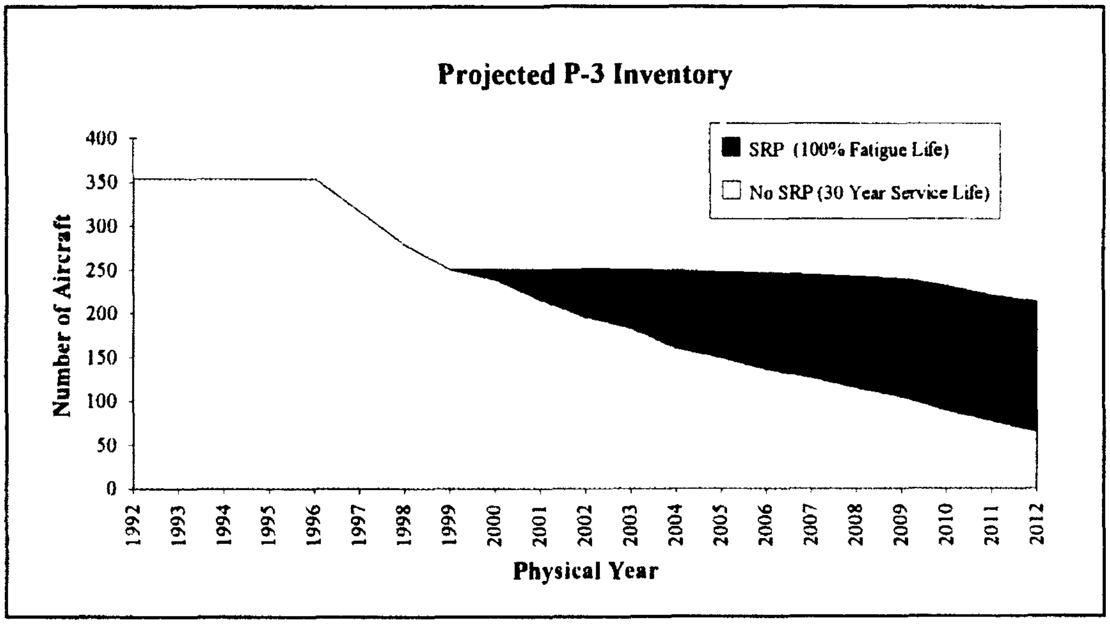

Figure 3 Projected P-3C Inventory Levels Assuming 100% SRP Capture Rate

The SRP is funded beginning in 1994 with the first aircraft being inducted into the program in 1996. There is a two year lag in funding so, for example, aircraft due for induction in 1996 are budgeted and paid for in 1994. The funding is currently approved through 1999 with a total of 46 aircraft budgeted. This relatively small number of aircraft funded for induction over the next 6 years creates a problem for program managers. The procurement schedule of the late 1960s and early 1970s has created a "bow-wave" of aircraft in need of the SRP. It will not be possible to induct all of these aircraft; some aircraft will have to be retired due to corrosion problems because there will be insufficient funding or capacity at the SRP facility. Consequently, a decision must be made as to which aircraft should be inducted into the SRP, when they should be inducted and which aircraft should be allowed to retire. MPAMOD makes these decisions taking into account aircraft age, fatigue life remaining and mission effectiveness, over a 20 year horizon.

3. Service Life Extension Program

The SRP's goal is to capture 100% of an aircraft's fatigue life. Another program, which may be considered in the future, is the Service Life Extension Program (SLEP). It is intended to increase the fatigue life of an aircraft beyond 100%. The idea of a SLEP is not new to military aircraft; many aircraft from all services have the capability of undergoing a SLEP which essentially gives an aircraft a new life. The airframe is "rebuilt" and sent back to the fleet as a new aircraft. A SLEP for P-3s has never been employed but is an option under consideration for future implementation. It is not expected that a P-3 SLEP would produce an aircraft that is as good as new, but

would rather add a fixed number of years to the aircraft's life. MPAMOD's implementation of the SLEP is such that any aircraft entering SLEP, regardless of its age or structural condition, will leave SLEP with a fixed amount of life remaining which is the same for all aircraft. The cost of the SLEP may vary for different aircraft, say SRP versus non-SRP aircraft, but there are no structural differences in the aircraft when they leave. Good cost estimates are not currently available for a P-3 SLEP program nor is an estimate of how much life could be added to the aircraft. The SLEP is included in MPAMOD so that when or if these number becomes available, program planners will have a tool to evaluate the cost-effectiveness of a SLEP.

B. REPLACEMENT AIRCRAFT

The use of the SRP and possibly a SLEP to extend the P-3 airframe life are no more than stop-gap measures to buy time to procure a new aircraft. If the MPA force is to exist in the future, a new aircraft must be procured at some point. In 1989 the sole provider of P-3s (Lockheed) shut down it's production line, but it was later reopened for the production of P-3s for foreign military sales. The Navy is now seeking funding in the POM 96 budget for the procurement of an upgraded version of the P-3 to be produced by Lockheed on the existing P-3 line. This new aircraft would have greater range and payload capabilities than the current P-3 and would also include an updated avionics suite. If approved, the plans call for the delivery of the first aircraft in 2001.

The production campaign, as implemented in MPAMOD, is described by specifying the production line opening and closing years along with minimum and maximum annual

production limits for each year the line is open. A cumulative production profile may also be defined for each year of the campaign. Annual budget limits must be set which cover R&D, line startup and shutdown costs as well as individual aircraft production costs. Finally, a production lag is set, specifying the amount of time between when an aircraft is paid for and when it arrives in inventory.

C. AVIONICS UPGRADE

The procurement of a new aircraft with state of the art avionics would greatly enhance mission effectiveness, but under the best of circumstances, it would be well after the turn of the century before there would be enough in inventory to significantly affect fleet capabilities. Consequently, some of the existing P-3s must be upgraded to ensure they are capable of accomplishing a more technically demanding mission. A limited Anti-Surface Warfare (ASUW) upgrade program has been approved to begin in 1994. Over a five year period from 1994 to 1999 sixty-eight aircraft will have their ASUW systems upgraded. The number of upgrades per year is fixed as is the budget. What must be done is to identify the candidate aircraft to be modified.

There are currently four variants of the P-3C aircraft: Update I, Update II, Update II.5 and Update III. The variants are distinguished by differences in tactical avionics equipment. Because of the nature of the ASUW upgrade kit, only the P-3C Update IIIs are capable of receiving the upgrade. Further, there are two types of Update IIIs, those that came off the production line as Update IIIs (production UIIIs) and those which were converted from baseline P-3C (non-production UIIIs). The production UIIIs are the

newest aircraft in inventory and the non-production UIIIs are the oldest. It would be nice to put all 68 upgrade kits in the newest aircraft but with only 34 production Update IIIs in inventory some of the kits must be installed in older aircraft. With such a small number of upgrade kits available, it is imperative to pick the best aircraft to upgrade, that is, those with the most life remaining.

MPAMOD addresses the technological effectiveness of the fleet through the use of minimum mission effectiveness goals. Aircraft with inherently different mission capabilities are assigned different mission effectiveness coefficients, a rating from 0 to 1, which represents the aircraft's relative capability in each of two mission areas, ASW and ASUW. For instance, aircraft with the ASUW upgrade, non-upgraded aircraft and new aircraft would each have different mission effectiveness coefficients for each mission area. New aircraft would presumably have the highest effectiveness and aircraft with the ASUW upgrade would have a higher ASUW effectiveness than non-upgraded aircraft. The mission effectiveness coefficient, indexed by year, decreases over time to account for the decrease in mission effectiveness which may result from an increase in the adversaries' capabilities as technological advances are made.

Minimum average mission effectiveness goals, for each mission area by year, are set by the program planners. In order to meet these goals MPAMOD will procure new aircraft where possible and more importantly choose the existing aircraft to update which, over the model's planning horizon, will provide the greatest benefit to the mission effectiveness of the fleet.

III. MODEL DESCRIPTION

The MPA modernization model will determine a schedule for procuring new aircraft, sending aircraft to SLEP and SRP, updating avionics and retiring old aircraft, subject to annual inventory goals, fleet-wide high-technology and average age goals and budget limitations. The problem is formulated as a production/inventory model where aircraft in inventory are moved over time in yearly increments. The model will minimize the total dollar expenditure over the model's planning horizon subject to the following general categories of constraints:

- Inventory flow balance constraints which allow for production, conversion via SRP and SLEP, aging and retirement of aircraft,
- Minimum and maximum inventory levels by year,
- A minimum average mission effectiveness goals by year and mission area,
- A minimum average life remaining goals for the inventory by year,
- Minimum and maximum line capacities for the new aircraft production facility, SRP, SLEP and avionics upgrade facilities,
- Minimum and maximum annual budget limits

Any mix of the SRP, SLEP, avionics upgrade program and new aircraft procurement programs can be included in a model run. Each program has associated with it annual budget constraints. It is possible to aggregate some or all of these individual budget

constraints into a single constraint to determine how to best allocate the overall modernization budget.

A. INDEX USAGE

1. Indexes

g	Aircraft cohort group (aircraft with similar age profiles: service acceptance year and fatigue life remaining)
t	Planning year
o	Operational mission
k	Successive SDLM number for an aircraft

2. Basic Index Sets

G	Existing cohort groups.
T	Planning years (fiscal) t of the model.
K	Possible SDLM numbers k for an aircraft ($k = 1^{st}, 2^{nd}, 3^{rd}, \ldots, K^{th}$).
O	Operational mission (ASW,ASUW).
$G_{kt}^{SDLM} \subseteq G$	Cohort groups of aircraft, which have not been inducted in the SRP or SLEP, due for their k^{th} SDLM in year t.
$G_t^S \subseteq G$	Cohort groups of aircraft eligible for SRP induction in year t. The age of the aircraft in year t must be within the user-defined SRP induction window.
$T^P \subseteq T$	Possible production campaign years.
$T^S \subseteq T$	Possible SRP program years.
$T^U \subseteq T$	Possible avionics update campaign years.

$T^L \subseteq T$	Possible SLEP campaign years.
$T_g \subseteq T$	Possible years t where aircraft from group g, which have not undergone SLEP or SRP, can exist in inventory. This subset of T is the years where the current age of the aircraft in year t is less than or equal to the maximum age for the group.
$T_g^U \subseteq T$	Possible years t where aircraft from group g with updated avionics, which have not undergone SLEP or SRP, can exist in inventory. This subset of T is the years where the avionics update facility is open or has previously been open, and the current age of the aircraft in year t is less than or equal to the maximum age for the group.
$T_g^6 \subseteq T$	Possible years t where SRP aircraft from group g can exist in inventory. This subset of T is the years where the SRP program is active or has previously been active, and the current age of the aircraft in year t is less than or equal to the maximum age for SRP aircraft for the group.
$T_g^{6U} \subseteq T$	Possible years t where SRP aircraft from group g with updated avionics can exist in inventory. This subset of T is the years where both the SRP and avionics programs are active or have previously been active, and the current age of the aircraft in year t is less than or equal to the maximum age for SRP aircraft for the group.
$T_{t'_t}^L \subseteq T$	Possible years t where aircraft which underwent SLEP in year t' can exist in inventory. This subset of T is the years where t is greater than the SLEP year (t') and less than or equal to the end of the planning horizon ($\lvert T \rvert$) or the SLEP year plus the life extension, whichever is less.
$T_g^P \subseteq T$	Possible years where new aircraft can exist in inventory. This subset of T is the years where the production facility has been open long enough for the first aircraft to be produced and enter the operational inventory.
$K_t^S \subseteq K$	Possible SDLM numbers k, for an SRP aircraft, in year t.

B. MODEL DATA

1. Budget Limits

B_t^P Annual budget for procurement of new aircraft.

B_t^{OM} Annual budget for operating and maintenance expenses and aircraft retirements.

B_t^S Annual Sustained Readiness Program budget.

B_t^{SDLM} Annual SDLM budget.

B_t^L Annual Service Life Extension Program budget.

B_t^U Annual budget for the avionics upgrade program.

2. New Aircraft Production

t_o^P Opening year of the new aircraft production line.

t_c^P Closing year of the new aircraft production line.

t_l^P Lag time in years between the time an aircraft is purchased and when it arrives in the fleet for use.

$\underline{P}_t$ Minimum number of aircraft that must be procured during year t ($t \in T^P$).

$\overline{P}_t$ Maximum number of aircraft which can be procured during year t ($t \in T^P$).

$\underline{P}_t^{\Sigma}$ Minimum cumulative number of aircraft required to be procured by year t ($t \in T^P$), in order to meet the contractual requirements.

c_t^P Unit cost of procuring a new aircraft in year t ($t \in T^P$).

c_t^{FP} Fixed cost of operating the production facility in year t. The startup and shutdown costs of the program are included as fixed costs.

3. Avionics Upgrade

t_o^U	Planning year t in which the update facility opens.
t_c^U	Planning year t in which the update facility closes.
$\underline{U}_t$	Minimum number of aircraft that must be upgraded during year t ($t \in T^U$).
$\overline{U}_t$	Maximum number of aircraft which can be upgraded during year t ($t \in T^U$).
$\underline{U}_t^{\Sigma}$	Minimum cumulative number of aircraft required to be upgraded by year t ($t \in T^U$), in order to meet the contractual requirements.
c_t^U	Unit cost of updating an aircraft in year t ($t \in T^U$).
c_t^{FU}	Fixed cost of operating the avionics upgrade facility in year t. The startup and shutdown costs of the program are included as fixed costs.

4. Sustained Readiness Program

t_o^S	Opening year of the SRP facility.
t_c^S	Closing year of the SRP facility.
$\underline{S}_t$	Minimum number of aircraft that must be inducted into the SRP during year t ($t \in T^S$).
$\overline{S}_t$	Maximum number of aircraft which can be inducted into the SRP in year t ($t \in T^S$).
$\underline{S}_t^{\Sigma}$	Minimum cumulative number of aircraft required to be inducted into the SRP by year t ($t \in T^S$), in order to meet the contractual requirements.
c_g^S	Cost of inducting an aircraft from cohort group g into the SRP.

c_t^{FS} Fixed cost of operating the SRP facility in year t. This cost also includes the startup and shutdown costs for the facility.

5. Service Life Extension Program

t_o^L Opening year of the SLEP facility.

t_c^L Closing year of the SLEP facility.

t^L Number of years aircraft life is extended as a result of SLEP.

$\underline{L}_t$ Minimum number of aircraft that must be sent through SLEP during year t ($t \in T^L$).

$\overline{L}_t$ Maximum number of aircraft which can be sent through SLEP in year t ($t \in T^L$).

L_t^{Σ} Minimum cumulative number of aircraft required to be sent through SLEP by year t ($t \in T^L$), in order to meet the contractual requirements.

c_g^L Cost of sending an aircraft from cohort group g through SLEP.

c_t^{FL} Fixed cost of operating the SLEP facility in year t. This cost also includes the startup and shutdown costs for the facility.

6. Inventory Data

I_g^0 Initial number of aircraft in cohort group g.

$\underline{I}_t$ Minimum desired number of aircraft in inventory each year. Included in the count to meet this minimum inventory are aircraft which may not be available for operational use, such as aircraft undergoing avionics upgrade, SRP induction, SLEP induction and SDLM.

$\bar{I}_t$ Maximum desired number of aircraft in inventory each year.

7. Operating and Maintenance Data

c_t^O	Unit cost of operating and maintaining a non-updated aircraft in planning year t.
c_t^{OU}	Cost of operating and maintaining an updated aircraft (avionics upgrade) in planning year t.
c_t^{OP}	Cost of operating and maintaining a new aircraft in planning year t.
a_k^{SDLM}	Average age of an aircraft undergoing its k^{th} SDLM.
c_k^{SDLM}	Average cost of the k^{th} SDLM.

8. High-technology / Mission Effectiveness Data

e_{ot}	High-technology effectiveness coefficient for an aircraft for mission area o in year t.
e_{ot}^U	High-technology effectiveness coefficient for an aircraft with updated avionics for mission area o in year t.
e_{ot}^P	High-technology effectiveness coefficient for a new aircraft for mission area o in year t.
$\underline{e}_{ot}$	Minimum average high-technology effectiveness desired for mission area o in year t. The average effectiveness is computed across all aircraft in inventory including those undergoing avionics upgrade, SDLM, SLEP and SRP induction in the given year.

9. Retirement and Aircraft Age Data

$\bar{a}_g$	Mandatory retirement age for aircraft from group g which have not been inducted into the SRP or SLEP.
$\bar{a}_g^S$	Mandatory retirement age for SRP aircraft from group g.

$\bar{a}^P$ — Expected retirement age for new aircraft. This data is used to compute the expected life remaining of a new aircraft.

r_{gt} — Remaining life as of year t for aircraft from group g which have not undergone any sort of life extension (SRP or SLEP). This data is computed as the difference between the maximum aircraft age ($\bar{a}_g$) and the aircraft's current age in year t.

r_{gt}^S — Remaining life as of year t for an SRP aircraft from group g. This data is computed as the difference between the maximum age for SRP aircraft ($\bar{a}_g^S$) and the aircraft's current age in year t.

$r_{t't}^P$ — Remaining life as of year t for new aircraft procured in year t' ($t > t'$). This value is computed as the difference between the expected retirement age for a new aircraft ($\bar{a}^P$) and the number of years since the aircraft entered the inventory.

$r_{t't}^L$ — Remaining life as of year t for aircraft which underwent SLEP in year t' ($t > t'$).This value is computed as the difference between the SLEP length (t^L) and the number of years since the SLEP occurred.

$\underline{r}_t$ — Minimum desired average life remaining in year t.

c_t^R — Unit cost of retiring an aircraft in year t.

C. DECISION VARIABLES

The decision variables can be categorized as inventory, retirement, SRP, SLEP, avionics upgrade and new aircraft procurement variables. All variables are general integers.

1. Inventory Variables

I_{gt}	Inventory of group g aircraft in year t which have not undergone an avionics upgrade or been inducted into the SRP or SLEP.
I^{U}_{gt}	Inventory of group g aircraft with updated avionics in year t.
I^{S}_{gt}	Inventory of group g aircraft in year t which have previously been inducted into the SRP.
I^{SU}_{gt}	Inventory of group g aircraft with updated avionics in year t which have previously been inducted into the SRP.
$I^{L}_{t't}$	Inventory of SLEP aircraft in year t which underwent SLEP in year t'.
$I^{UL}_{t't}$	Inventory of SLEP aircraft with updated avionics in year t which underwent SLEP in year t'.
I^{P}_{t}	Inventory of new aircraft in year t.

2. Retirement Variables

R_{gt}	Number of group g aircraft, which have not undergone SLEP, avionics upgrade or been inducted into the SRP, retired in year t.
R^{U}_{gt}	Number of group g aircraft with updated avionics retired in year t.
R^{S}_{gt}	Number of group g SRP aircraft retired in year t.
R^{SU}_{gt}	Number of group g SRP aircraft with updated avionics which are retired in year t.
$R^{L}_{t't}$	Number of SLEP aircraft retired in year t which were sent through SLEP in year t'.

$R^{UL}_{t't}$ Number of SLEP aircraft, with updated avionics, retired in year t which were sent through SLEP in year t'.

3. Avionics Update Variables

U_{gt} Number of aircraft from group g which received the avionics upgrade in year t.

U^S_{gt} Number of group g SRP aircraft which received the avionic upgrade in year t.

4. SRP Variables

S_{gt} Number of group g aircraft inducted into the SRP in year t.

S^U_{gt} Number of group g aircraft with upgraded avionics inducted into the SRP in year t.

5. SLEP Variables

L_{gt} Number of group g aircraft sent through SLEP in year t.

L^U_{gt} Number of group g aircraft with updated avionics sent through SLEP in year t.

L^S_{gt} Number of SRP aircraft from group g sent through SLEP in year t.

L^{SU}_{gt} Number of group g SRP aircraft, with upgraded avionics, sent through SLEP in year t.

6. New Aircraft Production Variables

P_t Number of new aircraft procured in year t.

D. CONSTRAINTS

The constraints are a mix of standard constraints and elastic constraints. The elastic constraints are denoted by $\dot{\leq}$. The variables used to elasticize each constraint are defined

as continuous variables. So while MPAMOD is referred to as an integer program throughout the thesis, technically it is a mixed integer model.

Elasticity allows constraints to be violated at a cost per unit of violation. This avoids the problem of the model being infeasible. Instead, the violated constraints can be studied by the modeler to gain further insight into the cause of the violation.

$$R_{gt} + U_{gt} + S_{gt} + I_{gt} = I_g^0 \qquad \forall\, g \in G,\ \ t = 1 \qquad (1)$$

$$I_{gt-1} - R_{gt} - U_{gt} - S_{gt} - I_{gt} - L_{gt} = 0 \qquad \forall\, g \in G,\ t \in T_g \qquad (2)$$

$$U_{gt-1} + I^U_{gt-1} - R^U_{gt} - S^U_{gt} - L^U_{gt} - I^U_{gt} = 0 \qquad \forall\, g \in G,\ t \in T^U_g \qquad (3)$$

$$S_{gt-1} + I^S_{gt-1} - R^S_{gt} - U^S_{gt} - L^S_{gt} - I^S_{gt} = 0 \qquad \forall\, g \in G,\ t \in T^S_g \qquad (4)$$

$$S^U_{gt-1} + U^S_{gt-1} + I^{SU}_{gt-1} - R^{SU}_{gt} - L^{SU}_{gt} - I^{SU}_{gt} = 0 \qquad \forall\, g \in G,\ t \in T^{SU}_g \qquad (5)$$

$$\sum_{g \in G} \left(L_{gt} + L^S_{gt}\right) - I^L_{t\,t+1} = 0 \qquad \forall\, t \in T^L \qquad (6)$$

$$\sum_{g \in G} \left(L^U_{gt} + L^{SU}_{gt}\right) - I^{UL}_{t\,t+1} = 0 \qquad \forall\, t \in T^L \qquad (7)$$

$$I^{L}_{t'\,t-1} - R^{L}_{t'\,t} - I^{L}_{t'\,t} = 0 \qquad \forall\ t' \in T^{L},\ t \in T^{L}_{t't} \qquad (8)$$

$$I^{UL}_{t'\,t-1} - R^{UL}_{t'\,t} - I^{UL}_{t'\,t} = 0 \qquad \forall\ t' \in T^{L},\ t \in T^{L}_{t't} \qquad (9)$$

$$P_{t-t^{P}_{l}} + I^{P}_{t-1} - I^{P}_{t} = 0 \qquad \forall\ t \in T^{P} \qquad (10)$$

$$\underline{P}_{t} \leq P_{t} \leq \overline{P}_{t} \qquad \forall\ t \in T^{P} \qquad (11)$$

$$\underline{P}^{\Sigma}_{t} \leq \sum_{t^{P}_{o} \leq t' \leq t} P_{t'} \qquad \forall\ t \in T^{P} \qquad (12)$$

$$\underline{S}_{t} \leq \sum_{g \in G^{S}_{t}} \left(S_{gt} + S^{U}_{gt}\right) \leq \overline{S}_{t} \qquad \forall\ t \in T^{S} \qquad (13)$$

$$\underline{S}^{\Sigma}_{t} \leq \sum_{t^{S}_{t} \leq t' \leq t} \ \sum_{g \in G^{S}_{t}} \left(S_{gt'} + S^{U}_{gt'}\right) \qquad \forall\ t \in T^{S} \qquad (14)$$

$$\underline{L}_{t} \leq \sum_{g \in G} \left(L_{gt} + L^{S}_{gt} + L^{U}_{gt} + L^{SU}_{gt}\right) \leq \overline{L}_{t} \qquad \forall\ t \in T^{L} \qquad (15)$$

$$\underline{L}^{\Sigma}_{t} \leq \sum_{t^{L}_{t} \leq t' \leq t} \ \sum_{g \in G} \left(L_{gt'} + L^{U}_{gt'} + L^{S}_{gt'} + L^{SU}_{gt'}\right) \qquad \forall\ t \in T^{S} \qquad (16)$$

$$\underline{U}_t \dot{\leq} \sum_{g \in G} \left(U_{gt} + U_{gt}^S \right) \leq \overline{U}_t \qquad \forall\ t \in T^U \qquad (17)$$

$$\underline{U}_t^{\Sigma} \dot{\leq} \sum_{t_o^U \leq t' \leq t} \ \sum_{g \in G} \left(U_{gt'} + U_{gt'}^S \right) \qquad \forall\ t \in T^U \qquad (18)$$

$$\underline{I}_t \dot{\leq} \left(\begin{array}{c} \sum_{g \in G} \left(\begin{array}{c} I_{gt} + I_{gt}^U + I_{gt}^S + I_{gt}^{SU} \\ + \\ U_{gt} + U_{gt}^S + S_{gt} + S_{gt}^U \\ + \\ L_{gt} + L_{gt}^S + L_{gt}^U + L_{gt}^{SU} \end{array} \right) \\ + \\ \sum_{t_o^L < t' < t} \left(I_{t't}^L + I_{t't}^{UL} \right) + I_t^P \end{array} \right) \dot{\leq} \overline{I}_t \qquad \forall\ t \in T \qquad (19)$$

$$\sum_{g \in G} \left(\begin{array}{c} \left(\begin{array}{c} (e_{ot} - \underline{e}_{ot}) \left(\begin{array}{c} I_{gt} + I_{gt}^S + \sum_{t_o^L \leq t' \leq t} I_{t't}^L \\ + \\ S_{gt} + L_{gt} + L_{gt}^S \end{array} \right) \\ + \\ (e_{ot}^U - \underline{e}_{ot}) \left(\begin{array}{c} I_{gt}^U + I_{gt}^{SU} + U_{gt} + U_{gt}^S \\ + \\ S_{gt}^U + L_{gt}^U + L_{gt}^{SU} + \sum_{t_o^L \leq t' \leq t} I_{t't}^{UL} \end{array} \right) \end{array} \right) \\ + \\ (e_{ot}^P - \underline{e}_{ot}) I_t^P \end{array} \right) \dot{\geq} 0 \qquad \forall\ t \in T,\ o \in O \qquad (20)$$

$$\left(\sum_{g \in G} \left(\begin{array}{c} (r_{gt} - \underline{r}_t)(I_{gt} + I_{gt}^{U} + U_{gt}) \\ + \\ (r_{gt}^{S} - \underline{r}_t)(I_{gt}^{S} + I_{gt}^{SU} + S_{gt} + S_{gt}^{U} + U_{gt}^{S}) \\ + \\ (r_{tt}^{L} - \underline{r}_t)(L_{gt}^{S} + L_{gt}^{SU} + L_{gt} + L_{gt}^{U}) \end{array} \right) \begin{array}{c} \\ + \\ \sum_{t_o^P \le t' \le t-L} (r_{t't}^{P} - \underline{r}_t) P_{t'} \\ + \\ \sum_{t_o^L \le t' \le t} (r_{t't}^{L} - \underline{r}_t)(I_{t't}^{L} + I_{t't}^{UL}) \end{array} \right) \dot{\geq} 0 \qquad \forall\, t \in T \tag{21}$$

$$\underline{B}_t^S \;\dot{\leq}\; \sum_{g \in G} c_g^S (S_{gt} + S_{gt}^U) + c_t^{FS} \;\dot{\leq}\; \overline{B}_t^S \qquad \forall\, t \in T \tag{22}$$

$$\underline{B}_t^L \;\dot{\leq} \sum_{g \in G} c_g^L \left(L_{gt} + L_{gt}^S + L_{gt}^U + L_{gt}^{SU} \right) + c_t^{FL} \;\dot{\leq}\; \overline{B}_t^L \qquad \forall\, t \in T \tag{23}$$

$$\underline{B}_t^U \;\dot{\leq}\; \sum_{g \in G} c_t^U (U_{gt} + U_{gt}^S) + c_t^{FU} \;\dot{\leq}\; \overline{B}_t^U \qquad \forall t \in T \tag{24}$$

$$\underline{B}_t^P \;\dot{\leq}\; c_t^P P_t + c_t^{FP} \;\dot{\leq}\; \overline{B}_t^P \qquad \forall\, t \in T \tag{25}$$

$$\underline{B}_t^{OM} \dot{\leq} \left(\begin{array}{c} \sum_{g \in G} \left(\begin{array}{c} c_t^{O}(I_{gt} + I_{gt}^{S}) + c_t^{OU}(I_{gt}^{U} + I_{gt}^{SU}) \\ + \\ c_t^{R}(R_{gt} + R_{gt}^{U} + R_{gt}^{SU} + R_{gt}^{S}) \end{array} \right) \\ + \\ c_t^{OP} P_t^{I} + \sum_{t_o^L < t' < t} \left(c_t^{O} I_{t't}^{L} + c_t^{OU} I_{t't}^{UL} \right) \\ + \\ \sum_{t_o^L < t' < t} c_t^{R} \left(R_{t't}^{L} + R_{t't}^{UL} \right) \end{array} \right) \dot{\leq} \overline{B}_t^{OM} \qquad \forall\, t \in T \qquad \textbf{(26)}$$

$$\underline{B}_t^{SDLM} \dot{\leq} \left(\begin{array}{c} \sum_{k} \sum_{g \in G_{kt}^{SDLM}} c_k^{SDLM}(I_{gt} + I_{gt}^{U} + U_{gt}) \\ + \\ \sum_{k \in K_t^S} \sum_{g \in G} c_k^{SDLM}(S_{gt-a_k^{SDLM}} + S_{gt-a_k^{SDLM}}^{U}) \end{array} \right) \dot{\leq} \overline{B}_t^{SDLM} \qquad \forall\, t \in T \qquad \textbf{(27)}$$

Constraints (1) through (10) define the inventory flow balance conditions of the model. Separate inventory balance constraints are maintained for each of the following "classes" of aircraft: SRP aircraft, SRP with updated avionics, SLEP aircraft, SLEP with updated avionics, aircraft which have not undergone SLEP or SRP life extensions and aircraft with updated avionics which have not been through SRP or SLEP. Aircraft are separated this way for two reasons: first, SRP, SLEP and non-life extended aircraft all have different retirement ages and second, keeping track of aircraft with updated avionics allows the "average high-technology" level of the fleet to be computed for each mission area.

Constraints (1) through (9) are complicated versions of standard production/inventory balance constraints with no demand. They state that the inventory

at the end of year t for a particular class of aircraft equals the inventory at the end of year t-1 plus any conversions of other classes of aircraft bringing aircraft into this class less aircraft which are retired or converted to become part of another class. For example, constraints (3) are the inventory balance constraints for aircraft with updated avionics which have not been inducted into the SRP or SLEP. These constraints ensure that the number of aircraft in inventory in year t (I^U_{gt}) is equal to the number in inventory in year t-1 (I^U_{gt-1}) plus the number which receive the avionics upgrade in year t-1 (U_{gt-1}) less aircraft which are retired (R^U_{gt}), inducted into the SRP (S^U_{gt}) or inducted into the SLEP (L^U_{gt}) in year t.

Constraints (1) bring the initial inventory of aircraft in each group into the model in year one. Constraints (2) are the balance constraints for aircraft which have not been inducted into the SRP or SLEP or received the avionics upgrade. Constraints (3), (4) and (5) are balance constraints for aircraft with updated avionics, SRP aircraft, and SRP aircraft with updated avionics, respectively. Constraints (6), (7), (8) and (9) define the balance conditions for SLEP aircraft. A separate inventory is maintained for every SLEP campaign year. This is done to keep track of when aircraft are "SLEPed" which in turn determines when they must be retired. Constraints (6) and (7) allow SRP and non-SRP aircraft to be sent through SLEP in year t. Constraints (6) apply to aircraft without updated avionics and (7) are for updated aircraft. The grouping differentiation (g) is discarded when an aircraft enters SLEP. This reduces the size of the model without sacrificing model realism. The group definitions provide aircraft retirement information, but when an aircraft completes SLEP it has a fixed amount of life remaining (t^L)

regardless of its group affiliation. The aircraft's group g is no longer required; however, the aircraft does become affiliated with a new group defined by the year the aircraft entered SLEP. All aircraft SLEPed in year t' must be retired within the next t^L years. For this reason the balance constraints (8) and (9), which ensure that once an aircraft is sent through SLEP it can either remain in inventory or be retired, only exist for t^L years past the SLEP year. Constraints (10) are simple production/inventory constraints, without demand, for new aircraft. There is a lag of t_i^P years from the time an aircraft is purchased (production is started) and when it arrives in inventory, i.e., production beginning in year t-t_i^P does not become part of inventory until year t.

Constraints (11) through (18) define the facility capacity limitations and contract requirements for the production of new aircraft, SRP inductions, SLEP inductions and avionics upgrades. The facility capacity constraints (11), (13), (15) and (17) are all similar in structure. These two-sided constraints describe the minimum and maximum activity limits for their respective programs. The lower limits model the minimum activity required to keep the facilities open, that is, the minimum number of aircraft which must be produced or minimum number of SRP or SLEP inductions that must take place in a given year to keep the facilities open. These limits are elastic, with the belief that a program can be shutdown and a facility closed for a cost. The upper limits model the physical capacity of the facilities and are therefore inelastic. Constraints (12), (14), (16) and (18) are cumulative activity constraints for the production of new aircraft, SRP inductions, SLEP inductions and avionics upgrades. These elastic constraints suggest that between the time when a facility opens and the current year, a minimum amount of

activity must take place to meet the contract requirements. For example, constraints (12) suggest that the total number of aircraft produced between the year the facility opens t_o^P and the current year t should be at least $\underline{P}_t^{\Sigma}$.

Constraints (19) suggest that the total number of aircraft in inventory in a given year must fall between a minimum and maximum level. The total inventory count includes aircraft being inducted into the SRP and SLEP and those receiving the avionics upgrade. This total also includes new aircraft once they reach the inventory t_I^P years after procurement.

Constraints (20) suggest that the average high-technology level for each mission area averaged across the entire fleet should be above the user defined level $\underline{e}_{ot}$. The constraints are written in their present form to preserve linearity. They are derived from the more natural form

$$\frac{e_{ot}\begin{pmatrix}\textit{aircraft without}\\ \textit{updated avionics}\end{pmatrix} + e_{ot}^{U}\begin{pmatrix}\textit{aircraft with}\\ \textit{updated avionics}\end{pmatrix} + e_{ot}^{P}(\textit{new aircraft})}{\textit{total inventory}} \geq \underline{e}_{ot}.$$

The high-technology level is a measure of aircraft capability in a given mission area. New aircraft, aircraft with updated avionics and non-updated aircraft are considered to have different mission capabilities and therefore each of these types of aircraft are given different high-technology ratings. These constraints encourage the model to consider the mission effectiveness of an aircraft when deciding which aircraft to send through the life extension programs and which to retire. The constraints also encourage the model to look

at aircraft age when deciding which aircraft should receive the avionics upgrade and tend toward upgrading those aircraft with the most life remaining.

Constraints (21) suggest that the average life remaining for the fleet should be greater than a minimum number of years $\underline{r}_t$. With the SRP, SLEP and non-life extended aircraft all having different expected operating lifetimes, "average life remaining" provides a good measure of fleet age. With a limited time horizon model such as this, these constraints play a critical role in forcing the model to generate a solution which is sensitive to aircraft age. The constraints prevent the model from generating a solution in which a large portion of the fleet must be retired shortly after the end of the model's planning horizon. As with the high-technology constraints, these constraints are written in a form to preserve linearity.

Constraints (22) through (27) define the budget limits for the SRP, SLEP, avionics upgrade program and procurement of new aircraft as well as the operating and maintenance budget and SDLM budget. These constraints suggest that there is a lower limit on spending as well as an upper limit. Defining a "budget band" such as this helps dampen large fluctuations in spending from year to year which might occur in the generation of an optimal solution. The SRP, SLEP, avionics upgrade and aircraft procurement budget constraints all have a fixed cost term (c_t^F). These fixed costs account for any annual non-recurring costs which may be associated with the program and also startup and shutdown costs incurred outside the active program years. Constraints (22) suggest that the recurring and non-recurring SRP costs in year t should fall between a minimum and maximum level. Similarly constraints (23), (24), and (25) model the

budget limits for the SLEP, avionics upgrade and procurement of new aircraft. Constraints (26) suggest that the operating and maintenance (O&M) costs and aircraft retirement costs should fall within the desired levels. The annual O&M costs can include any number of costs which are accumulated on a per aircraft basis such as consumables (fuel and oil), spares and support equipment. The budget limits for depot level maintenance (SDLM) are expressed in constraints (27). The annual budget accounts for the individual aircraft costs of performing a SDLM and does not address non-recurring costs associated with operating the facility. An "average" SDLM schedule is used to determine when an aircraft is due for SDLM. The schedule consists of the SDLM number for an aircraft, (1^{st}, 2^{nd}, 3^{rd},..., k^{th}), the average age of aircraft when they reach this k^{th} SDLM (a_k^{SDLM}) and the average cost of the k^{th} SDLM (c_k^{SDLM}). Any aircraft in inventory when it reaches a SDLM age automatically incurs the associated cost. Aircraft being inducted into the SRP during the same year that they are due for SDLM do not incur SDLM costs. The exclusion of SDLM costs for SRP inductees encourages the model to schedule the SRP inductions to coincide with SDLM due dates. Once an aircraft is inducted into the SRP it begins a new SDLM cycle, and therefore, from a SDLM perspective, the aircraft is new. This means that aircraft which were inducted into the SRP a_k^{SDLM} years previous to the current year t will incur a cost for their k^{th} SDLM.

E. OBJECTIVE FUNCTION

The objective of the model is to minimize the total cost over the model's entire planning horizon of SRP, SLEP, avionics upgrade and new aircraft procurement costs plus operating and maintenance, retirement and SDLM costs. The penalties for violating the elastic constraints are also included in the objective function.

$$
Min \sum_{t} \left(\begin{array}{c}
\sum_{g \in G} c_g^S (S_{gt} + S_{gt}^U) + \sum_{g \in G} c_g^L \left(L_{gt} + L_{gt}^S + L_{gt}^U + L_{gt}^{SU} \right) \\
+ \\
c_t^P P_t + \sum_{g \in G} c_t^U (U_{gt} + U_{gt}^S) \\
+ \\
\sum_{g \in G} \left(c_t^O (I_{gt} + I_{gt}^S) + c_t^{OU} (I_{gt}^U + I_{gt}^{SU}) \right) + c_t^{OP} P_t^I \\
+ \\
\sum_{t_o^L < t' < t} \left(c_t^O I_{t't}^L + c_t^{OU} I_{t't}^{UL} \right) \\
+ \\
\sum_{g \in G} c_t^R (R_{gt} + R_{gt}^U + R_{gt}^{SU} + R_{gt}^S) + \sum_{t_o^L < t' < t} c_t^R \left(R_{t't}^L + R_{t't}^{UL} \right) \\
+ \\
\left(\begin{array}{c}
\sum_{k} \sum_{g \in G_t^{SDLM}} c_k^{SDLM} (I_{gt} + I_{gt}^U + U_{gt}) \\
+ \\
\sum_{k \in K_t^{SRP}} \sum_{g \in G} c_k^{SDLM} (S_{gt - a_k^{SDLM}} + S_{gt - a_k^{SDLM}}^U)
\end{array} \right) \\
+ \\
\textit{Constraint Violation Penalties}
\end{array} \right)
$$

IV. IMPLEMENTATION AND RESULTS

MPAMOD is implemented in the General Algebraic Modeling System (GAMS) [Ref. 5]. GAMS is a high level language suitable for formulating large-scale optimization problems, which can employ a number of commercially available solvers. The X-System solver [Ref. 6] was selected for use with MPAMOD because of its proven ability to handle similar models such as PHOENIX and the Drash model. The model was developed and tested on the AMDAHL 5990-500 at the Naval Postgraduate School and has also been successfully run on a 486-50 (Intel 80486 CPU running at 50 MHz) micro-computer.

A typical scenario is set over a 20-year planning horizon with 247 aircraft "separated" into 75 cohort groups. Some mix of SRP, SLEP, avionics upgrade and new aircraft procurement programs are active. The resulting IP (ignoring continuous elastic variables) has about 4000 constraints and 12,000 variables with approximately 95,000 non-zero elements. Although, MPAMOD is an IP, inventory variables can be, and are in this implementation, defined as continuous variables. With the SRP, SLEP, avionics upgrade and retirement variables required to be general integers, the flow-balance constraints in the model will force the inventory variables to take on integer values.

A. COMPUTATIONAL EXPERIENCE

The definition of cohort groups is critical to the model in terms of model size, solution time and the detail of the schedule produced by the model. Individual aircraft, identified by bureau number, are "separated" into cohorts according to induction year and expected fatigue life remaining. The aircraft scheduled for SRP induction, SLEP induction, avionics upgrade and retirement each year are identified according to their cohort group. Therefore, the greater the number of groups, the more closely the schedule can be tied to individual aircraft. Carrying this to the extreme, a single aircraft per group, would result in scheduling aircraft, by bureau number, over a 20-year horizon. This may sound appealing but there are two potential problems with this approach. First, the resulting model will be large with approximately 14,000 constraints and 40,000 binary variables. The second problem is more managerial in nature. It is not reasonable to schedule aircraft by bureau number, over a 20-year period, for avionics upgrades, induction into the SRP or SLEP, or even retirement. With such a long time horizon, there are too many unknowns for a plan of this detail to work. A balance must be struck between model size and the level of detail necessary for a long-range planning model. For the scenarios presented in this chapter, the inventory of 247 P-3Cs has been "separated" into 75 cohorts. The cohorts were defined using data from the 15 October 1992 Quarterly SAFE report [Ref. 9].

Computational results from three typical scenarios, solved on both the AMDAHL and 486-50 micro-computer, are reported in Table 1. Scenario 1 includes the SRP and avionics upgrade program only. Scenario 2 includes the SLEP along with the SRP and

avionics upgrade program and finally, Scenario 3 exercises all four modernization programs, the SRP, SLEP, avionics upgrade and procurement of new aircraft. Truly optimal solutions (0% optimality gap) were not achieved for any of the test scenarios, but very good solutions, with optimality gaps as low as .9%, were obtained.

Table 1 MPAMOD COMPUTATIONAL RESULTS

	Scenario 1		Scenario 2		Scenario 3	
Objective Function ($100M)						
Modernization Program Costs		114.132		143.269		210.318
Penalties		476.225		435.787		243.315
Total		590.357		579.056		453.633
Optimality Gap		0.9%		1.8%		4.4%
Resource Usage (CPU Seconds)	Amdahl	PC	Amdahl	PC	Amdahl	PC
Model Generation (GAMS)	45.2	207.2	61.1	281.1	61.9	284.7
Solver Time to Best Incumbent	41.3	190.0	111.8	514.3	407.8	1875.9
Model Size						
Rows		4465		4809		4842
Columns		10422		13420		13453
Non-zero Elements		67388		95948		96224

The CPU time required to find the best integer solution is listed in Table 1 as "Solver Time to Best Incumbent". The CPU resource requirements increase with each successive scenario as additional modernization programs are added. The optimality gaps also tend to increase. However, even with the largest and most comprehensive model, Scenario 3, the solution time is less than 8 minutes with an optimality gap of 4.4%.

The same three scenarios were run on an Intel 486-50 machine with the solution times listed in Table 1. Given the size and complexity of the model, it is a tribute to the X-System that the PC times are as quick as they are. With MPAMOD's intended use as

a long-range planning model, 30 minute solution times are not out of line. Even so, two heuristics have been developed to generate near-optimal solutions in possibly less time.

1. Two-Phase Heuristic

This first heuristic approximately solves the model in two stages, first as an LP and then, using information from the LP, as an IP. It was observed early on in model testing that a large number of variables naturally take on integer values when the LP relaxation is solved. In many cases entire cohort groups are integer over the entire planning horizon. The two-phase heuristic exploits this natural integrality by a) solving the LP relaxation of the original IP, b) fixing all resulting variables, which are integer for the entire cohort, at their LP-determined value, c) removing all balance constraints associated with the fixed cohorts, and d) solving the resulting model as an IP. In doing this, the size of the model is typically reduced to one-half to one-quarter of its original size. The solution time for the reduced IP is drastically shortened, apparently without sacrificing solution accuracy. Table 2 compares the results of the original model with this heuristic. Micro-computer solution times are not listed, but can be reliably estimated as 4.6 times longer than the Amdahl times. The greatest gains are realized in the third scenario where MPAMOD spends a significant amount of time in the integer enumeration. There are cases, such as Scenario 1, where the heuristic generates a better solution than MPAMOD using branch-and-bound. Passing this incumbent heuristic solution to MPAMOD to use as a starting point could be helpful in generating optimal solutions; however, this has not been tested.

Table 2 COMPUTATIONAL RESULTS FOR MPAMOD AND TWO HEURISTIC ALGORITHMS.

	Branch and Bound	Two-Phase Heuristic		Rounding Heuristic
SCENARIO 1				
Model Size		Phase 1	Phase 2	
Rows	4665	4665	1048	4665
Columns	10422	10422	3011	10422
Non-zero Elements	67388	67388	16611	67388
Resource Usage (CPU Seconds)				
Model Generation (GAMS)	45.2	51.3	33.9	450.5
Solver Time to Best Incumbent	41.3	54.4*	2.7	65.0
Solution Accuracy	0.9%		0.8%	0.8%
SCENARIO 2				
Model Size		Phase 1	Phase 2	
Rows	4809	4809	2056	4809
Columns	13420	13420	5806	13420
Non-zero Elements	95948	95948	37697	95948
Resource Usage (CPU Seconds)				
Model Generation (GAMS)	61.1	64.5	45.2	607.6
Solver Time to Best Incumbent	111.8	81.9*	18.3	126.1
Solution Accuracy	1.8%		1.8%	1.8%
SCENARIO 3				
Model Size		Phase 1	Phase 2	
Rows	4842	4842	2518	4842
Columns	13453	13453	6769	13453
Non-zero Elements	96224	96224	44366	96224
Resource Usage (CPU Seconds)				
Model Generation (GAMS)	61.9	69.3	49.9	629.5
Solver Time to Best Incumbent	407.8	125.6*	57.5	179.1
Solution Accuracy	4.4%		5.6%	5.0%

* Solution time for the LP relaxation (Phase 1).

2. Rounding Heuristic

The rounding heuristic, as its name implies, uses a rounding technique to generate integer values rather than the traditional branch-and-bound technique. The model is solved as a sequence of LPs, with a rounding step between successive solves. The original model is solved as an LP and all variables corresponding to planning year 1 are rounded and fixed at integer values. Then, the partially fixed model is solved as an LP and the resulting variables for planning year 2 are rounded and fixed. The new, more restricted model is again solved as an LP and this sequence of solves, rounds, and restrictions is repeated until the entire planning horizon of the model has been covered.

The results for this heuristic are presented in Table 2. The results generally compare favorably to MPAMOD using branch-and-bound and the two-phase heuristic. One should not be put off by the apparently high CPU times, as better than 80% of this is attributable to GAMS overhead. GAMS must regenerate the entire model after each successive solve, a rather lengthy process which could be reduced with some special links between GAMS and the X-System. Consequently, focusing on the "Solver Time to Best Incumbent", the rounding heuristic produces good integer solutions in a reasonable amount of time.

There are limitations to the rounding heuristic due to the "unintelligent" rounding of variables. The rounding currently takes place without regard for the magnitude of the penalty values set for violating elastic constraints. "Unintelligent" rounding can increase the magnitude by which constraints are violated in comparison to the IP solution thereby resulting in excessively high penalties for the heuristic solution.

This might be improved by taking penalties into account when rounding the variables, but has not been done as part of this research.

B. DATA

A typical 20-year planning scenario requires a modest amount of input data once the cohort groups are defined. Annual budget limits must be set for each active program as well as annual facility minimum and maximum capacities. Total inventory level goals are set for each year along with minimum desired mission effectiveness by mission area and minimum average live remaining aspirations. Data for the SRP, avionics upgrade program and new aircraft procurement program was obtained from the P-3 Program Office (PMA-240) along with estimated annual O&M costs and SDLM data. In the absence of an actual SLEP for the P-3, artificial data describing a hypothetical program was created to test and demonstrate the capabilities of MPAMOD; this data does not necessarily resemble data for any programs being considered. The mission effectiveness coefficients, describing the relative capabilities of different types of aircraft in each mission area, are somewhat subjective values but could be the result of a formal study. No such study was carried out and the values used are based on informed guesses by the author.

As mentioned earlier, many of the modernization requirements are expressed as goals rather than hard requirements with the relative priority of each goal set by the program planners. Experience with MPAMOD has shown that the solution to the MPA modernization problem will usually violate of one or more of the following requirements:

minimum mission effectiveness, average life remaining, budgets or inventory levels. The penalties for violating the different constraints are defined as part of the input data. There are currently no hard and fast rules for determining the correct magnitude for the penalties; it is a trial-and-error exercise. However, it might be possible to devise formal rules for determining the penalties.

C. RESULTS

The objective of the model is to minimize the total cost, over the planning horizon, of all modernization programs plus the penalties for violating constraints. A host of reports are generated to assist program planners in analyzing the resulting modernization plan which best meets the minimum cost goal. These reports include:

1. Total inventory level and composition by year,
2. Number of SRP inductions, SLEPs, avionics upgrades and new aircraft procurements by year,
3. Number of SRP inductions, SLEPs and avionics upgrades by cohort group each year,
4. Annual budget expenditures for the modernization programs, SDLM and O&M,
5. Annual aircraft retirements by cohort,
6. Desired versus actual mission effectiveness by mission area each year,
7. Average aircraft life remaining by year.

MPAMOD is intended to be a planning model and as such there are a number of scenarios which can be envisioned to demonstrate its capabilities. Two scenarios are presented next to demonstrate the flexibility and usefulness of the model as a planning

tool. The first scenario examines the effects of changing a basic assumption about the life of the P-3 airframe and the second compares two hypothetical SLEP programs.

A major assumption underlying much of the SRP planning and the projection of future inventory levels is that the P-3 must be retired at 30 years of age unless it is inducted into the SRP. The 30-year retirement age is an educated estimate made by structural engineers, but it is still just an estimate. Multiple runs of MPAMOD can be made, with varying retirement ages, and the results compared to determine the significance of this estimate.

Using a 20-year scenario with the SRP and avionics programs active, two runs of the model were made. The first run, the baseline scenario, used an expected retirement age of 30 years and the second was identical to the baseline scenario except that the 30-year retirement age assumption was increased to 32 years. As expected, budget expenditures did not change for the SRP or avionics upgrade programs nor did the utilization of facilities; what changed was the selection of aircraft to be inducted into the SRP. The additional two years of life allowed the SRP to capture many aircraft which would have otherwise been retired because of insufficient funding or capacity at the SRP facility. Figure 4 shows the total inventory level assuming a 30-year versus 32-year retirement age. The significant drop in inventory for the first scenario between 1998 and 2003, is spread out over a period of 11 years between 2000 and 2011 in the second scenario with most of the retirements occurring in 2010. This change is simply the result of modifying an assumption with no additional program assets expended or changes made.

The only increased spending occurred in the area of O&M costs, as would be expected with the higher inventory level.

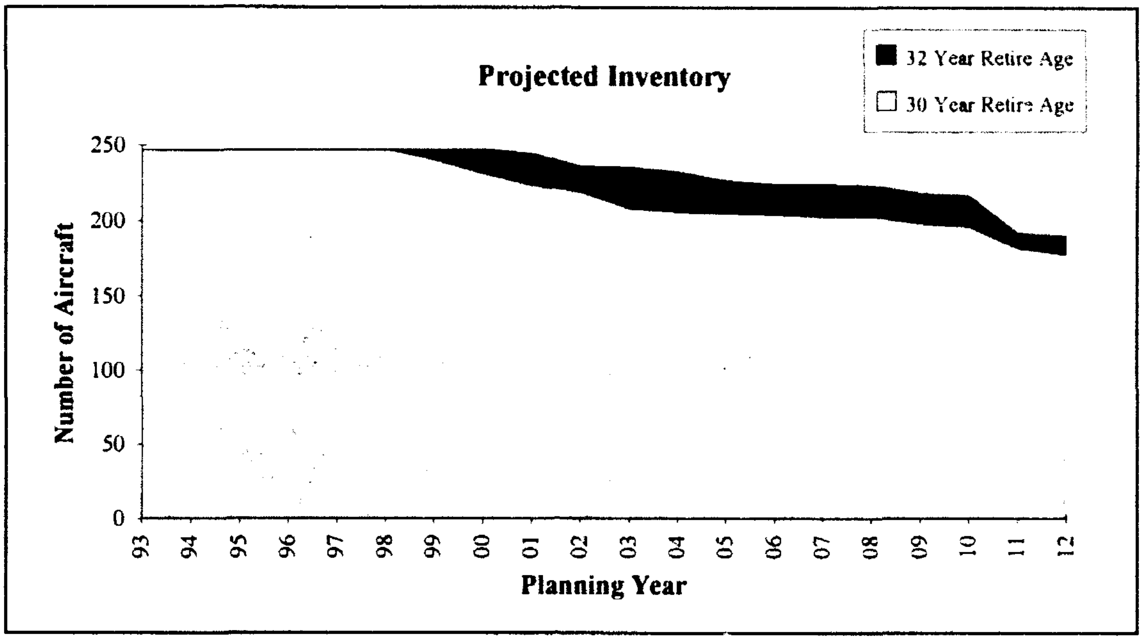

Figure 4 Projected P-3C Inventory Level (30 Year versus 32 Year Retirement Age)

The second demonstration of MPAMOD's capabilities compares two hypothetical SLEP programs. The first SLEP considered would extend aircraft life by 8 years for a cost of $20M per aircraft and the second would extend aircraft life by 16 years for a cost of $30M. Both programs are subject to the same opening and closing schedules and facility capacities. Comparable SLEP budgets are used to compare the two programs. That is, the budget for the second program is 50% greater than the first. A cursory analysis, focusing on the cost per year of life extension, might suggest that the 16 year/$30M program is more cost-effective than the 8 year/$20M program. However, when the two programs are compared, considering the existence of SRP and possible procurement of a new aircraft, the results are not obvious.

Multiple scenarios were run comparing the two SLEP programs where different mixes of the SRP and new aircraft programs were included. If a replacement aircraft is to be procured, then the MPA modernization goals can be met under the 8 year/$20M program with a lower overall cost than they can under the 16 year/$30M plan. With the 16 year/$30M program, a large number of SLEP aircraft are retired well before they reach the end of their SLEP-extended life. If the SLEP is used as a short-term program to buy time for a new aircraft to be introduced into the fleet, then 16 years of life extension is not needed to bridge the inventory gap when both the SRP and a replacement aircraft program exist. This result is arrived at under the limitation of a 20 year planning horizon, but it does not appear that extending the time horizon would significantly change the result. If it is assumed that the procurement of a new aircraft will continue past the end of the planning horizon then the SLEP aircraft will gradually retired and replaced with new aircraft and the SLEP length will have less effect on the age of the force. Of course, the preceding analysis is very narrow in scope and many potentially relevant factors have been ignored. However, this simple example demonstrates the capabilities of MPAMOD without getting into a comprehensive cost-effectiveness analysis of a SLEP program.

V. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

MPAMOD is an effective long-range planning model. The methodology is based on proven techniques which have been applied to similar military problems. The model provides the capability to evaluate proposed modernization programs and weigh the consequences of planning decisions when considered within the context of a comprehensive MPA modernization effort. The solutions are sufficiently accurate for long-range planning and the model lends itself well to "what-if" drills with relatively quick solution times. However, the accuracy and solution times can be improved. The two heuristics developed in conjunction with MPAMOD provide encouraging results in this effort to further reduce MPAMOD's solution time and achieve true optimality. The rounding heuristic produces integer solutions without solving the model as an IP, something that may be useful for large models where the solver has trouble obtaining a good integer incumbent The two-phase heuristic has proved most promising by significantly reducing the size of the model during the second phase and consequently near-optimal integer incumbent solutions are obtained much more quickly than with the full model. The utility of a long-range optimization planning model has been demonstrated in this thesis but the practicalities of implementing such a system for everyday use has been left for future work.

B. RECOMMENDATIONS

The successful implementation of the model will require some additional work. The user interface must be improved, both for data input and for the presentation of results. Making changes to the input data currently requires the user to directly modify the input file with a text editor. Unless the user has a working knowledge of GAMS and familiarity with the input file format, directly changing the file can easily lead to inadvertent errors being introduced. If the model is to be readily accepted and utilized, data entry must be easy and accurate. Some sort of full-screen, fill-in-the-blank, input program, which validates the data as it is entered, would greatly enhance the utility of the model. One such interface may already exist in the form of commercial spreadsheet programs. GAMS has been successfully interfaced with spreadsheet programs in the past [Ref. 5]. The output format should also be improved to better highlight the specific needs of the user. The reports currently generated by MPAMOD contain the pertinent results necessary to evaluate the modernization plan but as experience is gained with the model more effective reports can be created.

While a user-friendly interface will ease data input and help in the analysis of results, it is foreseeable that scenarios will arise which can not be handled by simply changing the data. For every scenario that the interface is set to handle, both for input and output, there are many others which have not even been thought of yet which the interface will not support. In these situations the model and /or data must be modified outside of the interface. This is not to say that making the model user-friendly is a bad

idea, but that the model will likely change and as much flexibility as possible should be designed into the interface.

MPAMOD generates good solutions in a reasonable amount of time, but it has not been possible to achieve, or at least prove, true optimality. Given the magnitude of the objective function in hundreds of millions of dollars, it would be worthwhile to reduce the optimality gap. The two heuristics presented earlier may provide help in this effort by quickly providing good incumbent solutions, but further work needs to be done on both of them. The rounding heuristic could be improved by taking into account the elastic constraint penalty values when rounding variables. Additionally, solution time could be reduced with some work on the GAMS/X-System interface, or by providing a much faster X-System interface procedure which would avoid interactions with GAMS entirely. The two-phase heuristic could be modified to better take advantage of the natural integrality of the relaxed, LP solution to MPAMOD. Currently, variables are fixed and constraints removed for the cohorts which are integer over the entire planning horizon, but nothing is done for the other groups. The LP solution could be used to tighten the bounds on some variables in the remaining groups and in certain cases it may be possible to fix the variables within a cohort for some contiguous set of years. For instance, if all of the LP optimal values for a cohort were integer for the first five years, these variables could be fixed at their LP optimal value prior to solving the model as an IP in phase 2.

C. ADDITIONAL APPLICATIONS

There are many naval air communities facing the same situation as MPA. The basic requirements for all of these different communities are similar: Inventory levels must support the forces structure requirements, budget limits must be adhered to, and mission effectiveness must be considered as well as the age of the fleet. MPAMOD is general enough in structure to be applied to other aviation communities with very little modification. If modifications are necessary, the GAMS implementation makes changes especially easy to incorporate.

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